

CURRENT PROBLEMS IN RESEARCH

Electrical Resistivity
at Low Temperatures

The pressure dependence of the electrical resistance of metals gives some clues about their Fermi surfaces.

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During this century there have been many investigations of the dependence of electrical resistivity of metals on temperature and pressure (1, 2). Broadly speaking, however, most of the temperature measurements have been made at atmospheric pressure, and most of the pressure measurements have been made at temperatures around room temperature. I would like here to describe some experiments which combine the two kinds of measurements and which are designed to find out how resistivity depends on pressure at low temperatures.

Why do we want this information? I shall first try to answer this question by contrasting the changes brought about by variations of temperature with those brought about by variations of pressure. We may think of a solid metal as consisting of a lattice of positive ions agitated by thermal vibrations and of an interpenetrating gas of conduction electrons. For many purposes we may discuss the metal in terms of its energy levels—for example, the energy levels associated with the normal modes of vibration of the lattice or the allowed kinetic energy levels of the conduction electrons. In general, if the volume does not change, these energy levels do not change, and an alteration of the temperature simply alters the distribution of the electrons or lattice

vibrations among these levels. But if the volume changes, then the levels themselves also change. Thus, changing the temperature at fixed volume acts as a method (albeit a rather gross one) of exploring how these levels are distributed; altering the volume is a means of changing them. A combination of the two methods therefore enables us both to change the levels and to investigate their properties after the change.

Having emphasized the different roles of temperature and pressure changes, I would now like to show that in certain ways they are closely related. It is a fact of experience that the thermal expansion coefficient of most substances is positive. Consequently, from the thermodynamic identity expressed in the equation:

$$\left(\frac{\partial V}{\partial T}\right)_p = -\left(\frac{\partial S}{\partial p}\right)_T \quad (1)$$

it follows that the entropy of such substances falls when they are isothermally compressed. But now it is well-known from thermodynamics that at constant pressure the entropy of a substance also falls when its temperature is lowered. We therefore see that in this respect an increase in pressure is similar to a decrease in temperature. Indeed, so attractive is this idea that G. N. Lewis (3) wished to use it as a basis for extending the third law of thermodynamics. His tentative statement of such an extension was as follows: "At all temperatures the entropy of a pure

crystal is zero at infinite pressure." The proposal was never in fact developed, presumably because the extrapolation to infinite pressure cannot usefully be made; whatever the formal similarity between increasing pressure and decreasing temperature, their actual physical effects are ultimately quite different (4). For more restricted purposes, however, this similarity is quite valid and very useful, as I now hope to show.

The equilibrium lattice properties of many metals can be discussed in terms of a reduced temperature $\tau = T/\theta$, where T is the actual temperature and θ is the characteristic lattice temperature of the metal (for example, the Debye temperature). θ , which ideally is independent of T , is related to the characteristic frequency of the lattice, and if the lattice is compressed its characteristic frequency, and hence θ , increases. Thus the reduced temperature may be decreased either by reducing the actual temperature or by increasing the pressure. This suggests that pressure coefficients and temperature coefficients must be related. Take, as an example, the lattice entropy S , which depends only on T/θ . Thus

$$\left(\frac{\partial S}{\partial T}\right)_V = \frac{S'}{\theta} \quad \left(S' \equiv \frac{dS}{d\tau}\right)$$

$$\left(\frac{\partial S}{\partial V}\right)_T = -\frac{S'}{\theta} \cdot \frac{T}{V} \cdot \frac{d \ln \theta}{d \ln V}$$

Therefore

$$\left(\frac{d \ln S}{d \ln V}\right)_T = \gamma \left(\frac{d \ln S}{d \ln T}\right)_V \quad (2)$$

Here I have introduced the parameter γ (which equals $-d \ln \theta / d \ln V$), called the Grüneisen parameter. We thus see that the volume coefficient of entropy is related to the temperature coefficient of entropy by the parameter γ . In fact, a thermodynamic transformation of this equation leads to Grüneisen's law of thermal expansion:

$$\frac{1}{V} \left(\frac{\partial V}{\partial T}\right)_p = \frac{\gamma \beta C_V}{V} \quad (3)$$

where C_V is the specific heat at constant volume, β is the compressibility, and $(1/V)[(\partial V/\partial T)_p]$ is the thermal expansion coefficient. This relationship then affords a means for determining γ (which measures the change of θ with

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